Ejercicio 1 – Convocatoria Ordinaria 1 – Curso 20/21

Dado el sistema de ecuaciones lineales $AX = b$ donde,

$$A = \begin{pmatrix} 1 & -2 & 0 \\ 0 & \lambda + 2 & -3 \\ \lambda + 2 & 0 & 0 \\ \lambda & 4 & -2 \end{pmatrix} \quad \text{y} \quad b = \begin{pmatrix} 0 \\ \lambda \\ 0 \\ 0 \end{pmatrix}$$

Discutir el sistema según los valores del parámetro λ y resolverlo cuando sea posible usando Cramer.

$$A = \begin{pmatrix} 1 & -2 & 0 \\ 0 & \lambda+2 & -3 \\ \lambda+2 & 0 & 0 \\ \lambda & 4 & -2 \end{pmatrix}$$

$$\begin{vmatrix} 1 & -2 & 0 \\ 0 & \lambda+2 & -3 \\ \lambda+2 & 0 & 0 \end{vmatrix} = 6(\lambda+2) = 0 \iff \lambda = -2$$

$$\begin{vmatrix} 1 & \boxed{\begin{matrix} -2 & 0 \\ 0 & -3 \end{matrix}} \\ 0 & \\ -2 & 4 \quad -2 \end{vmatrix}$$

$$\begin{vmatrix} 1 & -2 & 0 \\ 0 & 0 & -3 \\ -2 & 4 & -2 \end{vmatrix} = -12 + 12 = 0$$

$$rg(A) = \begin{cases} 3 & \ldots\ldots \ \lambda \neq -2 \\ 2 & \ldots\ldots \ \lambda = -2 \end{cases}$$

$$A|b = \begin{pmatrix} 1 & -2 & 0 & 0 \\ 0 & \lambda+2 & -3 & \lambda \\ \lambda+2 & 0 & 0 & 0 \\ \lambda & 4 & -2 & 0 \end{pmatrix}$$

$$|A|b| = \lambda \cdot \alpha_{24} = \lambda(-1)^{2+4} A_{24} = \lambda \cdot \begin{vmatrix} 1 & -2 & 0 \\ \lambda+2 & 0 & 0 \\ \lambda & 4 & -2 \end{vmatrix} =$$

$$= \lambda(-4(\lambda+2)) = -4\lambda(\lambda+2) = 0 \iff \lambda = 0, -2$$

$\lambda = 0$

$$A|b = \begin{pmatrix} 1 & -2 & 0 & 0 \\ 0 & 2 & -3 & 0 \\ 2 & 0 & 0 & 0 \\ 0 & 4 & -2 & 0 \end{pmatrix} \qquad \begin{vmatrix} 1 & -2 & 0 \\ 0 & 2 & -3 \\ 2 & 0 & 0 \end{vmatrix} = 12 \neq 0$$

$rg(A|b) = 3$ para $\lambda = 0$

$\lambda = -2$

$$A|b = \begin{pmatrix} 1 & -2 & 0 & 0 \\ 0 & 0 & -3 & -2 \\ 0 & 0 & 0 & 0 \\ -2 & 4 & -2 & 0 \end{pmatrix} \qquad \begin{vmatrix} 1 & -2 & 0 \\ 0 & 0 & -3 \\ -2 & 4 & -2 \end{vmatrix} = -12 + 12 = 0$$

$$\begin{vmatrix} -2 & 0 & 0 \\ 0 & -3 & -2 \\ 4 & -2 & 0 \end{vmatrix} = 8 \neq 0$$

$rg(A|b) = 3$ si $\lambda = -2$

En resumen:

$$rg(A|b) = \begin{cases} 4 \; \dots & \lambda \neq 0, -2 \\ 3 \; \dots & \lambda = 0, -2 \end{cases}$$

- Si $\lambda \neq 0, -2 \Rightarrow rg(A) = 3 \neq 4 = rg(A|b) \Rightarrow$ S.I.
- Si $\lambda = 0 \Rightarrow rg(A) = 3 = 3 = rg(A|b) = n^{\underline{o}}$ incogs $\Rightarrow$ S.C.I. $\Rightarrow$ S.C.D
- Si $\lambda = -2 \Rightarrow rg(A) = 2 \neq 3 = rg(A|b) \Rightarrow$ S.I.

$\underline{\underline{\lambda=0}}$

$$\left.\begin{array}{l} x - 2y = 0 \\ 2y - 3z = 0 \\ 2x = 0 \\ \cancel{4y - 2z = 0} \end{array}\right\} \quad \det\begin{pmatrix} 1 & -2 & 0 \\ 0 & 2 & -3 \\ 2 & 0 & 0 \end{pmatrix} = 12 \neq 0$$

$$x = \frac{\begin{vmatrix} 0 & -2 & 0 \\ 0 & 2 & -3 \\ 0 & 0 & 0 \end{vmatrix}}{|A|} = \frac{0}{12} = 0 \qquad y = \frac{\begin{vmatrix} 1 & 0 & 0 \\ 0 & 0 & -3 \\ 2 & 0 & 0 \end{vmatrix}}{|A|} = \frac{0}{12} = 0$$

$$z = \frac{\begin{vmatrix} 1 & -2 & 0 \\ 0 & 2 & 0 \\ 2 & 0 & 0 \end{vmatrix}}{|A|} = \frac{0}{12} = 0$$

Solución: $\underline{\underline{(0,0,0)}}$